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Short Communication

Synthesizing topological acoustic rainbow trapping at deep-subwavelength corners

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Topological boundary states, the smoking gun of topological insulators (TIs), have been widely investigated in a variety of settings [1,2]. Among the youngest members of this seemingly growing family, we have higher-order topological insulators (HOTIs) whose lower-dimensional topological corner states violate the conventional bulk-boundary correspondence [3]. Benefiting from cunningly contrived designs and the aid of 3D printing, sonic crystals (SCs) have served as ideal platforms to study topological properties in an acoustic context. Among them, prototypical models to achieve two-dimensional (2D) acoustic HOTIs include Su–Schrieffer–Heeger (SSH) model and quadrupole TI [4], where the degenerate corner states are observed within a single band gap and are typically characterized by a binary topological invariant (zero for trivial and one for nontrivial).

Along an equally exciting avenue, capturing multi-frequency waves at different spatial locations, known as rainbow trapping, has triggered abundant explorations in classical systems, such as mechanics [5], photonics [6], and acoustics [7,8]. Versatile related applications have been also discussed in the form of wavelength demultiplexing, frequency routing, wave switching, and energy harvesting [9]. Thanks to the robustness against flaws, topological rainbow trapping based on topological boundary states, has been theoretically proposed and experimentally reported in acoustics, photonics and mechanics [10–15]. In contrast, rainbow trapping through lower dimensional localized states, indeed possesses the capability to generate much stronger spatially selective sound energy localization, i.e., acoustic topological "hot spots" with higher quality factor, which is indispensable for the design of a high-efficient acoustic rainbow trapping device.

In this work, we introduce translational deformations of the units of a SC as the approach to synthesize acoustic HOTIs. Up to now, binary topological phases in HOTIs are generally modulated by altering the coupling strengths among their constituents (see Fig. 1a), which results in zero-energy corner states that remain degenerate and are pinned at one specific frequency. Here, we demonstrate that the nontrivial Zak phase induced by varying the translational deformation, has a continuous evolution as shown in Fig. 1b. The eigenfrequency spectra indicate that the bulk bands accompanied by a complete band gap are nearly unaffected. More importantly, the degeneracy of the corner states is lifted, and the sound energy can be confined separately at different frequencies. We take full advantage of this deformation approach, to engineer topological rainbow trapping at the corners of HOTIs, whose underlying synthesis permits deeply and spectrally broad subwavelength confinement. In particular, our two-lane implementation gives rise to directionally opposing, yet spatially selective, concentration of topological states.

We begin with presenting the synthetic space comprising a gradient topological phase evolution. As illustrated in Fig. 1c, the approach to obtain a continuous Zak phase evolution is based on engineering a square lattice that is composed of cavity-coupled waveguides. The lattice constant is $a = 5$ cm, and the height of the waveguide is $a/4$. The width and height of the single cavity are $a/2$ and a, respectively. The pristine unit cell (blue) is progressively deformed (yellow) by virtue of smoothly deforming, i.e., shifting the cavity by Δ_x and Δ_y (more intuitive deforming process is given in the Supplementary materials Note I). From the extremes, i.e., the corner of the synthetic deformation space, we design two SCs with the paramaters: $(\Delta_x, \Delta_y) = (0, 0)$ (blue) and $(\Delta_x, \Delta_y) = (0.5a, 0.5a)$ (yellow). Despite the fact that the band

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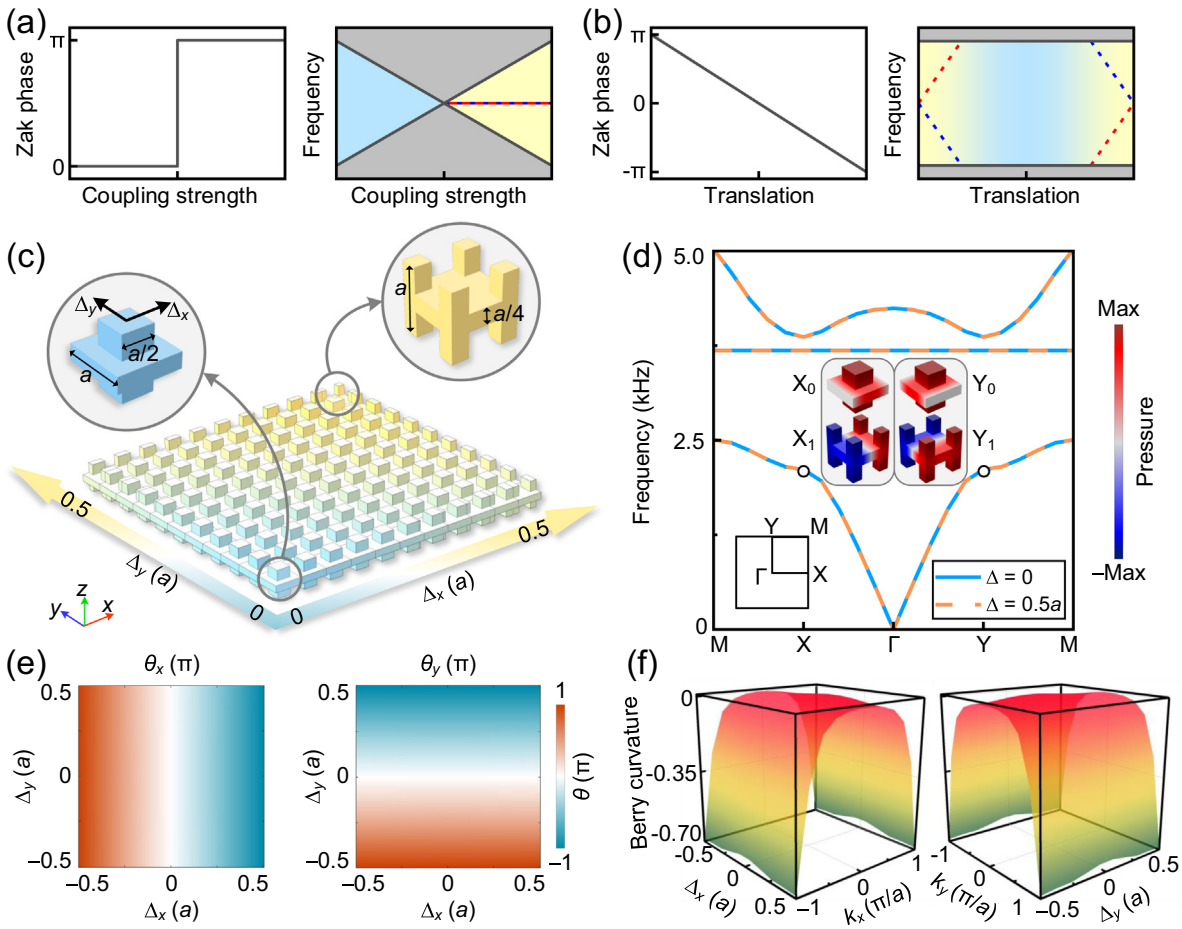

Fig. 1. (Color online) (a, b) Schematic diagrams of the Zak phase evolution and the eigenfrequency spectrum when (a) altering the coupling strength, and (b) varying the translational deformations of the units. Gray regions and the coloured lines represent the bulk bands and the corner states, respectively. (c) Schematic of the SC where the deformation of the cavity is continuously varied along the x and y directions. Insets: schematics of the primitive cells with $(\Delta_x, \Delta_y) = (0, 0)$ (blue) and $(\Delta_x, \Delta_y) = (0.5a, 0.5a)$ (yellow). (d) Band diagrams for the SCs with $\Delta = 0$ and $\Delta = 0.5a$. Insets show the corresponding acoustic eigenstates at the X (Y) point of the 1st BZ. (e) Zak phase contour versus the deformation parameters. (f) Berry curvature calculated in the synthetic space.

diagrams for these two SCs appear identical, as illustrated in Fig. 1d (the Supplementary materials Note II), the parities of their eigenstates at the high-symmetry positions of the first Brillouin Zone (BZ) yield differing Zak phases. As shown in Fig. 1e, the Zak phases for the first band are determined through a Wilson-loop approach [16]:

$$\theta_i(\Delta_x, \Delta_y) = \int_{-\pi}^{\pi} \mathrm{d}k_i A_i(\mathbf{k}), i = x, y. \tag{1}$$

Here, $A_i(\mathbf{k}) = i\langle u(\mathbf{k})|\partial_i|u(\mathbf{k})\rangle$ is the Berry connection of the Bloch function $|u(\mathbf{k})\rangle$, which is the periodic part of the Bloch eigenstates $\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}}|\psi_n(\mathbf{k})\rangle$ with $\mathbf{k} = (k_x, k_y, \Delta_x, \Delta_y)$. The Zak phases θ_x and θ_y undergo a closed-path loop with a -2π phase difference in the synthetic subspaces during the transforming process. As can be seen, the deformation Δ_y is almost irrelevant to θ_x, and Δ_x makes no impact on θ_y. Considering the deformation parameters as the synthetic periodic degrees-of-freedom, the Bloch eigenstates satisfy the periodic condition of $|\psi_n(k_x+2\pi/a, k_y+2\pi/a, \Delta_x+a, \Delta_y+a)\rangle = |\psi_n(k_x, k_y, \Delta_x, \Delta_y)\rangle$ in this 4D synthetic space. We demonstrate that the topology of this synthetic system can be characterized by the so-called second Chern number [17,18]:

$$C_2 = \frac{1}{32\pi^2}\int_{\mathrm{BZ}} \mathrm{d}\mathbf{k}\epsilon_{lmno}\mathrm{Tr}[F_{lm}(\mathbf{k})F_{no}(\mathbf{k})], \tag{2}$$

where ϵ_{lmno} is an anti-symmetric tensor of rank 4, and its subscripts stand for the four degrees of freedom $(x, y, \Delta_x, \Delta_y)$. $F_{lm}(\mathbf{k}) = \partial_l A_m(\mathbf{k}) - \partial_m A_l(\mathbf{k})$ represents the Berry curvature as calculated in Fig. 1f. Due to the symmetries of the Berry curvature, $F_{lm} = -F_{ml}$, and the independence from the (k_x, Δ_y) and (k_y, Δ_x) space, the second Chern number in Eq. (2) can be simplified to the integral of the Berry curvature in these two synthetic spaces, which results in $C_2 = 1$ for the first band [19]. Consequently, the existence of $(4-2)$D topological boundary states in the (Δ_x, Δ_y) subspace of the proposed synthetic TI, which are 0D corner states in real space, is ensured by the nonzero second Chern number.

In order to realize such topological corner states, we design a synthetic acoustic HOTI whose nontrivial interior (yellow), which is composed of units with translational deformation $\Delta_x = \Delta_y = \Delta \neq 0$, is enclosed by a trivial exterior (blue) of undeformed units, $\Delta = 0$ (see inset of Fig. 2a). The spectral evolution of the eigenstates along the synthetic axis Δ is depicted in the Supplementary materials Note III. It is found that the degeneracy of two off-diagonal corner states is lifted through smooth variation of Δ, which is the backbone of rainbow trapping at deep-subwavelength corners as we will discuss in the following. Yet, compared with the previously reported strategies to engineer acoustic HOTIs, our present scheme allows for a "pumping-like" approach to spectrally trap topological sound at spatially designated corners, where confinements about 21 times smaller than

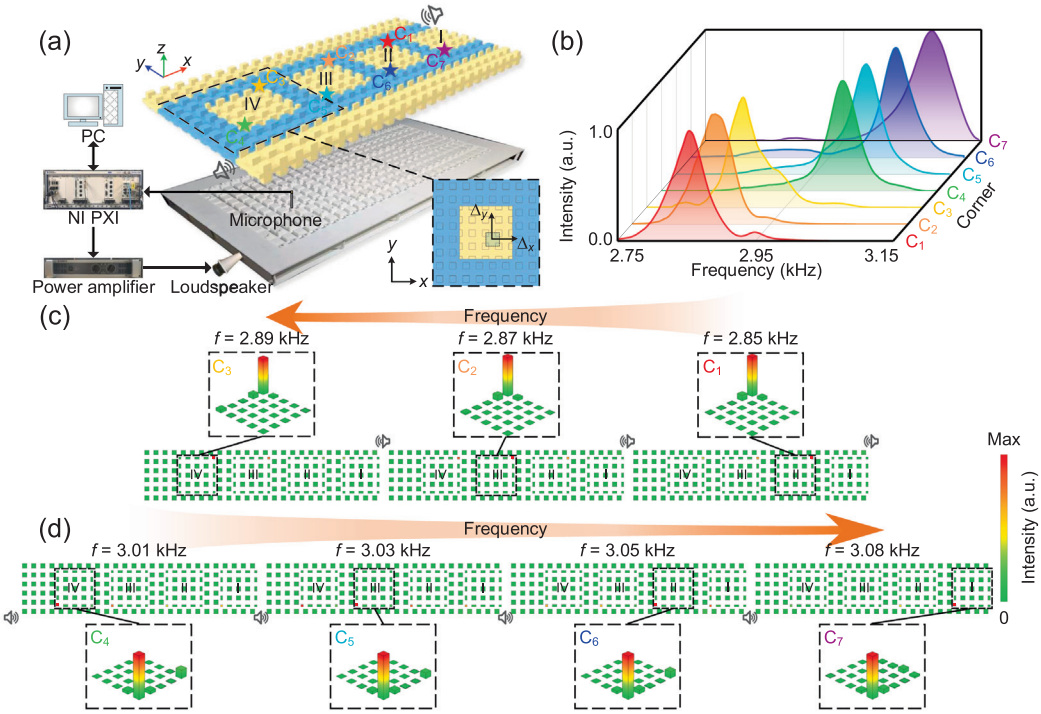

Fig. 2. (Color online) Topological rainbow trapping at corners in a two-lane highway. (a) Schematic and photo of the designed structures. (b) Detected sound intensities at different corners vs. scanning frequency. (c, d) Experimental sound intensities evaluated along the (c) backward and (d) forward going lanes at their respective spectral states.

the acoustic wavelength are possible. Also, we emphasize the superiority of our corner state localization in terms of quality factor and state volume, as opposed to those that stem from the topological edge states and SSH featured corner states (see the Supplementary materials Note IV and Note V), which holds promising prospects for a large variety of topological functional devices for energy harvesting and sensing. We further exemplify the strong robustness of such corner state against various defects in the Supplementary materials Note VI.

Taking advantage of the strong localization, we design and 3D print a finite planar array of HOTIs, each synthesized to host their very specific translational deformation according to the afore discussed evolution of corner states. We engineer parallel rainbow traps in the form oppositely growing spectral state lanes. As Fig. 2a showcases, we excite those trapped corner states via flanked interfaces that separate the trivial (blue) from the nontrivial (yellow) zone. Along them, in their respective unidirectional direction, we launch edge states whose spectral components are trapped at the proximate designated corner site. In Fig. 2a, the units in the nontrivial regions I to IV are displaced with the translation parameters varying from 0.46a to 0.49a in an identical interval of 0.01a, respectively. The spectral response of the flanked topological edge channels is intended to overlap with the working frequencies of the array of excited corner states (see the Supplementary materials Note VII). As a result, we expect to obtain acoustic rainbow trapping at the seven corners highlighted by the coloured stars in Fig. 2a according to the global eigenspectrum discussed in the Supplementary materials Note VIII. Hence, according to the edge channel dispersion, we spectrally sweep sound from a point source along this topological lane that will bridge the trivial region to sequentially couple with the first three corner states C_1–C_3. In the opposite direction, in a likewise manner, we excite the remaining consecutive four corner states C_4–C_7. Thus, as gathered in Fig. 2b, the experimental fingerprints of all corner state excitations involved in the topological rainbow trap are shown through a series of intensity spectra detected at the respective corners (more details can be found in the Supplementary materials Note IX). We expand upon this comprehensive presentation by showing the spectral response along the backward (Fig. 2c) and the forward (Fig. 2d) going lanes (see the Movie S1 of the Supplementary materials for the two-lane rainbow trap). At about a tenth of the acoustic wavelength compared to the cavity side length, we clearly see that the arrayed HOTIs sustain the targeted rainbow trapping of corner states along the respective one-way lane comprising its spectrally increasing window. Due to the inherent viscous losses in the experiments as shown in Fig. 2b, we obtain a small degree of overlap among the neighbouring corner states, yet the targeted ones always remain with the highest acoustic intensity.

In conclusion, we have built a synthetic acoustic TI by engineering translational deformations of the involved units in cavity-coupled waveguides. By doing so both in the x and y directions, we designed a 4D synthetic acoustic HOTI, where the non-degenerate topologically protected corner states are characterized by a nonzero second Chern number. Compared with the conventional SSH model, corner states in the translational transformed structure are much stronger localized, as characterized by the lower state volumes. Based on this, we designed a topological two-lane rainbow trap of corners states with superior localization of sound energy at deeply subwavelength scales. Our findings have demonstrated unparalleled potential for acoustic functional devices, useful for energy harvesting, enhanced sensing, and wave filtering.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

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Author contributions

Zhiwang Zhang initiated the project and conceived the idea. Zhiwang Zhang, Ying Cheng, Xiaojun Liu, and Johan Christensen guided the research. Danwei Liao conducted FEM simulations. Danwei Liao, Yixian Liu, and Zhiwang Zhang designed the experimental setup and conducted the measurements. Danwei Liao, Zhiwang Zhang, Ying Cheng, and Johan Christensen wrote the manuscript. All authors contributed to the discussions of the results and the manuscript preparation.

Appendix A. Supplementary materials

Supplementary materials to this short communication can be found online at https://doi.org/10.1016/j.scib.2023.07.016.

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