Acoustic field enhancement and subwavelength imaging by coupling to slab waveguide modes

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We present a theoretical study on the amplification of evanescent sound waves produced by coupling to trapped modes hosted by a fluidic planar waveguide. Total internal reflection at interfaces of different refractive indexes can be frustrated by the introduction of a slow slab waveguide which is leading to a gigantic field enhancement, useful for sensitive transducers and acoustic shock lithotripsy. The mechanism behind the evanescent field coupling that is also known as tunnelling barrier penetration in quantum mechanics is here adopted for its use in an acoustic superlens. The higher spatial harmonics produced by a subwavelength object can couple to trapped modes of the slow waveguide and be reproduced as an image at a distant plane. We suggest a practical implementation of these ideas by means of a silicone rubber slab containing positive acoustic wave propagation parameters. © *2010 American Institute of Physics*.
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Subwavelength imaging of sound by means of exotic metamaterial properties has been a topic of growing interest in the past decade. The milestone demonstration by Pendry that negative refraction can make a perfect lens[1] has sparked considerable interest in the pursuit of an acoustical counterpart. Although negative acoustic refraction has not been observed yet, the theoretical prediction of this effect in rubber spheres associated with Mie resonances can lead to demonstration of the first acoustic Veselago medium.[2–5] Acoustic filter components such as Helmholtz resonators, membranes, and open side branches in tubes are practical systems in order to tune the effective mass density and bulk modulus to negative values. Zhang *et al.*[6] proposed a planar lens claimed to posses a negative refractive index, although its focusing capabilities were limited by diffraction. Alternatively, Lee *et al.*[7,8] considered the combination of a membrane and an open side branch inside a tube, as a feasible design for achieving true double negativity. Interestingly, the component responsible for an effective negative bulk modulus, the open side branch, has been recently modified into an acoustic double-fishnet design constituting a bulk metamaterial for the suppression of sound propagation.[9] Other acoustic imaging devices which do not rely on negative refraction are planar hyperlenses that are converting evanescent fields associated to high resolution features into propagating waves and holey metamaterials for full three-dimensional imaging.[10,11]

In this letter, we present a theoretical analysis of a yet unseen lensing paradigm that is based on a simple slow fluid slab. Our design yields efficient sound transfer of both propagating and evanescent waves, provided that a slab waveguide resonance condition is met. The latter are related to higher spatial harmonics of a subwavelength image, that is, fields containing components of wave vectors exceeding those of propagating waves, which, for example, are also occurring under total internal reflection (TIR) at an interface or prism at oblique irradiation above a critical angle. We show that evanescent coupling to a slow waveguide gives rise to amplification of these short-range waves, which is the key ingredient for (bio)chemical sensing and detection applications such as ultrasonic shattering for medical use.[12,13]

Let us start by studying wave properties of a slab embedded into an acoustic heterostructure. The inset of Fig. 1 illustrates a fluid slab of index n_g surrounded by reference media of index $n=1$ symmetrically facing a coupling layer (prism) of index $n_c=5$ on either side, located at $z=d_1=0$ and $z=d_2=21h$, where h is the slab height and z runs perpendicular to the interfaces. What at first glance might seem trivial to the reader, will later become less evident due the fact that sound is being tunneled through the waveguide with high intensity, giving rise to full transmission. We present a quantum mechanical analogy, in which we assume that the acous-

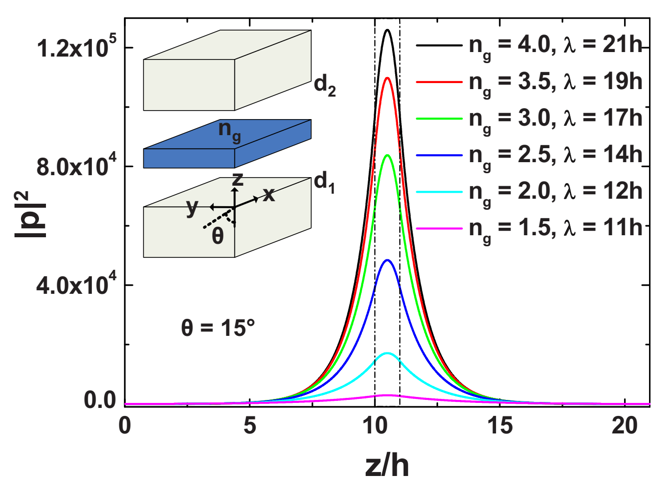

FIG. 1. (Color online) Sonic field enhancement assisted by coupling to the fundamental mode of a slab waveguide. At the coupler interface (lower part, $z/h=d_1=0$) an irradiating oblique sound beam ($\theta=15°$) leaks evanescent waves into the surrounding upper medium due to TIR. Calculations have been performed for guides of various values of n_g which correspond to different resonant wavelengths λ yielding a frustrated TIR which amplifies the decaying field. Inset: Diagram of the guide with index of refraction n_g and height h adjacent to the surrounding medium ($n=1$) which extends up/down to the coupler interfaces ($n_c=5$) located at d_1 and d_2. The mass density is assumed to be uniform in this structure.

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tic heterostructure contains a uniform mass density ρ but with the speed of sound $c(z)$ being a function of space. The slab is acting as a potential well characterized by a potential $V(z)=[1-n^2(z)]2\pi^2/\lambda^2$ and a particle of effective energy $E=(1-n_c^2\sin^2\theta)2\pi^2/\lambda^2$, where θ is the incidence angle with respect to z in the prism of refraction index n_c. The analogy consists in realizing that the pressure $p(z)$ in this system satisfies the same wave equation as the wave function $\psi(z)$ of a unit-mass particle subject to the noted potential, that is, the Schrödinger equation as follows:

$$-\frac{1}{2}\frac{d^2\psi(z)}{dz^2}+V(z)\psi(z)=E\psi(z). \quad (1)$$

Thus the potential well, like a planar slab waveguide, contains at least one trapped mode which can couple to acoustic radiation and that under TIR is decaying as an evanescent wave into the barrier ($n=1$). This wave carries no energy but at a given incidence angle ($\theta=15°$) and frequency of the sound approximately matching the parallel wave vector of the trapped mode, the intensity is amplified inside the gap region all the way up to the slab as can be seen in Fig. 1. For this particular tuned system, $|p|^2$ grows with increasing values of n_g that are associated to different wavelengths $\lambda(n_g)$ of the fundamental waveguide resonance. This strong layer guided field enhancement is a simple but robust scheme. When the incident sound is on resonance with a slab-trapped mode in Fig. 1, transmission occurs with 100% probability. This is a well established transmission mechanism in quantum mechanics, as shown by Chang, Esaki, and Tsu.[14] Therefore, the sound pressure has to take a large value inside the slab in order to compensate for the exponential decay up to the far coupler, in which it has to have the same value as the incident pressure to actually undergo full transmission. As one can see, changes in the index of refraction leads to a dramatical variation in the on-resonance field enhancement that could form the basis of a sensitive transducer measuring those changes induced by a biological specimen. Likewise, the strong resonance-frequency dependence on the dielectric environment is the key ingredient of many optical and plasmonic sensors.[15,16] This dramatical and localized pressure increase within a linearized acoustic regime might also be useful for the destruction of kidney stones when a fluid layer hosting trapped modes is inserted as an acoustic agent.

The fact that an evanescent field can be amplified when coupled to a slab of high index of refraction suggests the possibility of using the trapped modes for superlensing applications. The trapped modes are controlled by the acoustic frequency and the angle of incidence, which determine the amount of parallel wave vector that is coupling to the resonance. In other words, $c_0>c_\parallel>c_g$ is the inequality condition which governs the waveguide resonance, where the velocities of sound in the surrounding medium c_0, in the interfacial region $c_\parallel$, and in the waveguide c_g have been used. The amount of parallel momentum $k_\parallel$ is increased, which we here measure with the interfacial speed of sound $c_\parallel$ when the fluid slab is much slower compared to the surrounding $c_0 \gg c_g$. This determines the $k_\parallel$ spectrum confined within the sound cones of the surrounding medium and the slab that is needed to recover a subwavelength source in the image plane by means of high transmission of evanescent waves. Reminiscent to Pendry's perfect lens, we expect the recovery of subwavelength details to take place from a sound emitting

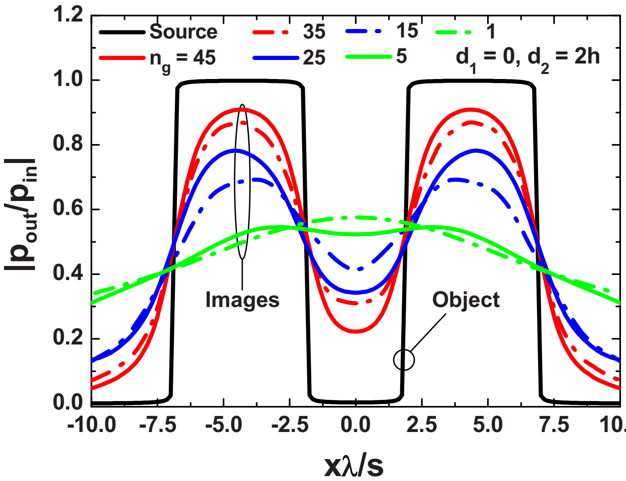

FIG. 2. (Color online) Source $|p_{\rm in}|$ at $d_1=0$ and images $|p_{\rm out}|$ at $d_2=2h$ (h is the slab height) produced by a slab waveguide for different values of n_g in the slab. With the chosen configurations the images are plotted for the second sonic slab guided mode at a fixed index of refraction corresponding to wavelengths five times larger than the slit width ($s=\lambda/5$).

source (two slits of width $s=\lambda/5$ in our example) at an image plane with a focal length $h/2$ that is similar to the distance of the source in front of the slab. But in contrast to Pendry's lens, the chosen slab indexes of refraction n_g are strictly positive, as indicated in Fig. 2. In the current problem, we use no coupling layer as the evanescent waves naturally are built up by the subwavelength source. Different values of n_g are giving rise to different spectral locations of the slab guided modes. As an example, we concentrate on the second symmetric guided mode and normalize the spatial coordinates to the resonant wavelengths and the slit width of the source, so that all plots can be mapped on the same graph. The rigorously coupled acoustic fields comprising the source can then be Fourier transformed and mapped onto the image plane (using the waveguide transmission coefficient) at $z/h=d_2=2$ as illustrated in Fig. 2. As expected, slabs with index of refraction near unity do not possess the ability to recover a near-field object into the image plane. However, when the speed of sound inside the guide is decreased toward extreme values up to $n_g=45$, almost undistorted images are obtained as shown in Fig. 2. It is important to stress that this paradigm of subwavelength imaging by means of trapped-mode excitation is not due to a localized cavity or particle resonance giving rise to negativities or channelling. Instead, our slow fluid layer acting as a waveguide does the job, whereupon extensions to other focal lengths and smaller objects can be worked out.

In order to make this concept far more than a theoretical idea we need to find realistic materials for the implementation. The initially proposed heterostructure for the field enhancement served as an tutorial example for understanding the amplification process and there the choice of the materials in not so crucial as for the case of the superlens. Thus for the proposed lens it is critical that we find a material in which the sound propagates very slowly compared to the surrounding. For the implementation we choose a silicone rubber layer with $\rho=1300$ kg/m^3 and $c=22$ m/s.[3] We note that the shear waves of the rubber layer due to the high velocity contrast to the background fluid can be neglected, such that this solid layer can be treated as a fluid. Again, we emphasize that this material possesses strictly positive acous-

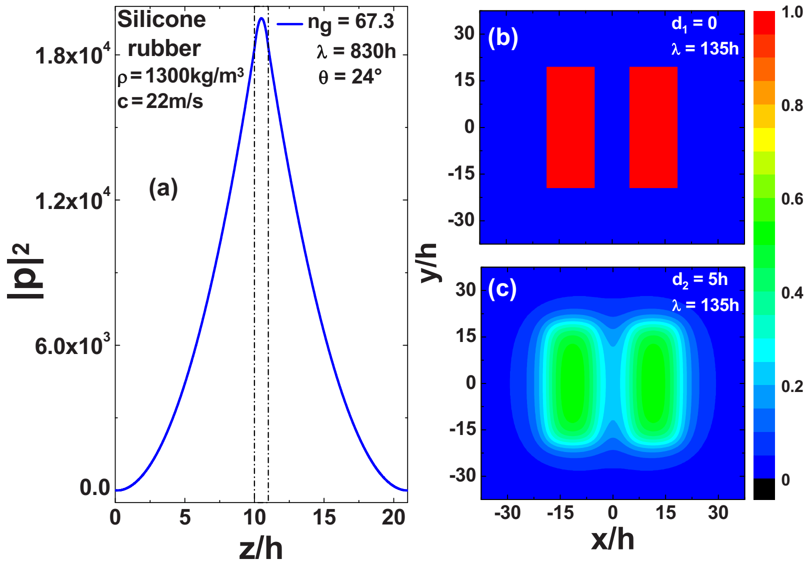

FIG. 3. (Color online) Realistic implementation of our schemes for field enhancement (left) and superlensing (right) by means of a planar silicone rubber (ρ=1300 kg/m^3, c=22 m/s) waveguide. (a) At an air/water interface (z/h=0) an incident sound beam undergoes TIR. The decaying wave is significantly amplified when it is coupled to the trapped modes of the silicone rubber slab of index n_g=67.3 relative to water. (b) Complex subwavelength source (slit width $s=\lambda/10$) placed at $z/h=d_1=0$. (c) Image plane at $z/h=d_2=5$ with highest field intensity $|p_{\mathrm{out}}|$=0.65 normalized to $|p_{\mathrm{in}}|$, as produced by the slab.

tical constitutive parameters. Recalling the sketch of the inset in Fig. 1, we now use an air/water interface in order to generate TIR, so that water fills the intermediate region surrounding the slab, and the couplers are just the air outside this structure. At an angle of θ=24° we are able to couple the evanescent wave undergoing TIR to the guided modes of the silicone rubber slab that in this case also is giving rise to a gigantic pressure enhancement, as shown in Fig. 3(a). With those chosen parameters, sound is trapped inside the guide at a wavelength that is significantly larger than the slab height h, namely, λ=830h. We also stress that the quantum mechanical analogy no longer applies when one has a space dependent mass density $\rho(z)$. Hence, we need to use the acoustic wave equation for spatial inhomogeneities,

$$\frac{d}{dz}\left[\frac{1}{\rho(z)}\frac{dp(z)}{dz}\right]+\frac{\omega^2 p(z)}{c^2(z)\rho(z)}=0. \quad (2)$$

In the same framework we consider a silicone rubber slab to be immersed in water and to function as a superlens for subwavelength imaging. We perform calculations considering both the source and the image to be located twice the slab height h away from the rubber lens. As Fig. 3(b) depicts, calculations have been performed for a source consisting of two slits of width $s=\lambda/10$. The subwavelength resolution capabilities of the nonmetamaterial slab are clearly demonstrated in the image of Fig. 3(c), in which two-thirds of the maximum source field intensity are recovered, which confirms the feasibility of slow fluid slabs not requiring resonating metamaterial elements, to function well as superlenses.

In conclusion, we have demonstrated that evanescent waves in a simple slab waveguide produce gigantic amplification of the incident wave pressure, with potential application to ultrasonic shattering and (bio)chemical sensing. We have used this amplification to propagate fine details of a source for deep-subwavelength imaging. The silicone rubber superlens that we present in Fig. 3(c) is an adequate device for underwater sonography operating in the audible range at a frequency of 1.1 kHz for a slab height of 10 mm.

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