Acoustic gain in piezoelectric semiconductors at $\varepsilon$-near-zero response

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We demonstrate strong acoustic gain in electric-field biased piezoelectric semiconductors at frequencies near the plasmon frequency in the terahertz range. When the electron drift velocity produced by an external electric field is higher than the speed of sound, Cherenkov radiation of phonons generates amplification of sound. It is demonstrated that this effect is particularly effective at $\varepsilon$-near-zero response, leading to giant levels of acoustic gain. Operating at conditions with strong acoustic amplification, we predict unprecedented enhancement of the scattered sound field radiated from an electrically controlled piezoelectric slab waveguide. This extreme sound field enhancement in an active piezo material shows potential for acoustic sensing and loss compensation in metamaterials and nonlinear devices.

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Active materials producing electromechanical coupling can be achieved by piezoelectric (PZ) structures but can also be realized by means of electrostriction and thermo-electromechanical effects. If a PZ material undergoes mechanical deformations, it produces electric charges, and vice versa, when an electric field is applied, the structure is mechanically strained. Commonly, PZ materials are found to be made of both natural and synthesized crystals, among which we can name ferroelectric oxides, quartz, Rochelle salt, and synthetic ceramics such as lead titanate, zinc oxide, and bismuth ferrite. Today there exist numerous versatile applications based on PZ sensors, actuators, and switches used for the automotive industry, medical instruments, and telecommunications. On a more fundamental aspect, PZ semiconductors play a crucial role, and it is believed that this discipline offers a plethora of yet unseen electromechanical effects within a high-frequency regime. Nanopiezotronics and nanopyroelectrics, for example, comprise the study of thermal and electromechanical properties associated with wurtzite-compound or ferroelectric polar, comprise the study of thermal and electromechanical properties associated with wurtzite-compound or ferroelectric polar.

Amplification of mechanical waves (sound) was observed by Hutson et al. in 1961 in a CdS semiconductor slab [10]. When an acoustic field, upon external irradiation, deforms the PZ material, space charges are generated by the elastic field and cause the electrons to redistribute accordingly. The electron drift induced by an external field can become supersonic, that is, $v_d > v_s$ [where $v_s(v_d)$ is the sound (electron drift) velocity], and amplification can take place due to the phonon emission of carriers [11–13]. In other words, acoustic gain is produced when the electron drift velocity exceeds the velocity of sound, which is in direct analogy to Cherenkov radiation. From the constitutive PZ relations, it can be shown that the electromechanical stress $T$ is inversely proportional to the dielectric constant $\varepsilon$ in the absence of a net charge in the semiconductor. The presence of acoustically generated charge carriers will, however, modify this result slightly. It then follows that elastic strains such as acoustic amplification can be boosted significantly when tuned toward $\varepsilon$-near-zero (ENZ) response.

In this Rapid Communication we present a seminumerical study of the amplification process of sound in PZ semiconductor materials. We distinguish between a low-frequency regime where acoustic gain unambiguously is explained by the emission of sound due to Cherenkov radiation and a high-frequency one where $\varepsilon$ approaches zero. In ENZ materials, light propagates with almost no phase advance due to the extended sizes of the wavelength [14,15]. This has been achieved by metamaterials and has resulted in prominent applications such as supercoupling and directive emission of light and sensing, to name a few [16]. In the context of PZ acoustic amplification at ENZ response, we show that gain can be many orders of magnitude larger compared to amplification caused by Cherenkov emission. In addition, we design an optomechanical device giving rise to enhanced acoustic radiation triggered by electrical switching with the cycle of half a wave round-trip.

Consider the constitutive relations for a piezoelectric material,

$$T = cS - \varepsilon E,$$

$$D = \varepsilon E + P + eS,$$

where $T$, $S$, $D$, $E$, $P$, $c$, $e$, and $\varepsilon$ are the stress, strain, electric displacement, electric field, spontaneous polarization, stiffness, piezoelectric $e$ constant, and permittivity, respectively. In cubic (zinc blende) structures the spontaneous polarization is zero, but in hexagonal (wurtzite) structures the spontaneous polarization is nonzero and is usually higher than the piezoelectric contribution to the electric displacement. In reality, the above equations are tensor equations for the crystal;
however, discarding field variations in space except along one coordinate, \( z \), the above scalar system suffices for the analysis.

The constitutive relations above are for isentropic conditions such that Onsager relations apply. This approximation will only be used in the constitutive relations, and losses are accounted for by using a finite and frequency-dependent complex carrier mobility. The elastic equation in one dimension is

\[
\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial z} = c \frac{\partial^2 u}{\partial z^2} - \rho \frac{\partial E}{\partial z},
\]

(2)

where \( \rho \) and \( \omega \) are the mass density and angular frequency, respectively, and \( S = \frac{\partial T}{\partial z} \), with \( u \) being the material displacement. We consider only electrons as the acoustic response of the much heavier holes can be discarded. The continuity equation reads

\[
\frac{\partial J}{\partial z} = -\frac{\partial \rho_e}{\partial t}, \quad J = q \mu_e n E + q F D_n \frac{\partial n_e}{\partial z},
\]

(3)

where \( J \) and \( \rho_e \) are the free-current density and the space-charge density, respectively. \( F \) denotes the fraction of acoustically generated electrons that are free to move, \( q \) is the elementary charge, \( \mu_e \) is the electron mobility, \( D_n \) is the electron diffusivity, and \( n = n_0 + n_s, n_r \), and \( n_0 \) are the total electron density, the generated acoustic electron density, and the electron density at equilibrium, respectively. Combining the above expressions implies an electromechanical dispersion relation:

\[
\rho \omega^2 = c k^2 - \frac{k^2 \omega^2}{\omega^2 + i \mu_e \varepsilon_0 k D_n} - \varepsilon,
\]

(4)

This equation is a fourth-order complex polynomial in \( k = \frac{\varepsilon}{\omega} - i \alpha \), where \( \alpha \) is the damping term. Now, since \( |\alpha| \ll \frac{\varepsilon}{\omega} \), we may safely, for small fields \( E_0 \), replace \( k \) by \( \frac{\varepsilon}{\omega} \) in the denominator of the second term on the right-hand side. The resulting dispersion equation is a second-order polynomial in \( k \) whose roots we denote \( k_1 \) and \( k_2 \) henceforth. The dispersion relation is supplemented by a Drude permittivity frequency response for semiconductors,

\[
\varepsilon = \varepsilon_{\infty} \left( 1 - \frac{\omega_p^2}{\omega^2 + \tau^{-2}} \right),
\]

(5)

where \( \omega_p \) is the plasmon frequency and the complex mobility \( \mu_n \) is

\[
\mu_n = \frac{\mu_{DC}}{\left( \frac{\tau^{-1}}{\omega} \right)^{-1} + i \omega},
\]

(6)

where \( \mu_{DC} \) is the dc mobility and \( \tau \) is the carrier collision time. We note that either the plasmon frequency or the collision time (or both) are in the terahertz (THz) range for many semiconductors so that the permittivity approaches zero and changes sign only in the range of THz frequencies. A strong mechanical response within this spectral range is predicted since vanishing permittivities lead to a tremendously high stress, \( T \sim 1/\varepsilon \), as derived from Eq. (1), and is responsible for obtaining enhanced gain or absorption, which we will see in the following. The above dispersion relation is solved for the case of zinc-blende InSb using the following parameters: \( \varepsilon = -0.07 \text{ C/m}^2 \), \( c = 4.7 \times 10^{10} \text{ Pa} \), \( m_{\text{eff}} = 0.014 \) (in units of the free-electron mass), \( \rho = 5770 \text{ kg/m}^3 \), \( n_0 = 2 \times 10^{22} \text{ m}^{-3} \), \( \mu_{DC} = 7.7 \text{ m}^2 \text{ s}^{-1} \text{ V}^{-1} \), \( \varepsilon_{\infty} = 15.7 \), \( \tau = \mu_{\text{eff}} \mu_{DC} / q \), and \( F = 1 \), corresponding to a plasmon frequency \( f = 2.71 \text{ THz} \). It is evident from the \( k_1 \) wave-number-plots in Figs. 1(a) and 1(b) for a frequency range far below the plasmon frequency \( \omega_p \) that an abrupt transition from absorption to gain occurs near the position where the Cherenkov condition is fulfilled, i.e., where the drift speed \( v_d = \mu E_0 \) surpasses the speed of sound \( v_s \). The small deviation in the transition frequency away from the Cherenkov condition stems from the appearance of the small diffusion term in Eq. (4). Gain (absorption) requires the real and imaginary terms of the wave vector to have the same (opposite) sign [refer to Fig. 1(b)].

In the case of InSb and the parameters above, the transition between absorption and gain takes place when the dc electric field equals approximately \(-430 \text{ V/m} \). It is also evident from Fig. 1(a) that the strength of the gain or absorption is weak since the damping term of \( k_1 \) is at most \( 0.005 \text{ m}^{-1} \), with increasing gain toward increasing electric field strength and low frequencies. In Figs. 1(c) and 1(d), we also plot the wave-number component \( k_2 \). Since the real and imaginary components are always of opposite sign irrespective of the dc electric field value, only absorption is possible for this mode, and \( k_2 \) excitations are always damped during propagation. Further, it can be seen that the absorption strength is rather weak for \( k_2 \) modes, a result that is similar to earlier absorption results [11].

Similar plots of \( k_1 \) and \( k_2 \) are shown in Fig. 2 for frequencies around the plasmon frequency. It follows from Eq. (4) that

\[
\varepsilon = 0 \quad \text{for a Drude permittivity response this occurs when } \omega_p^2 = \omega^2 - \tau^2, \text{ assuming } \omega_p > \tau, \text{ as is the case for, e.g., intrinsic InSb},
\]

the imaginary part of \( k \) can take on arbitrarily...
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large positive (or negative) values as the electric field is increased. Thus, we can tailor the intrinsic acoustic absorption or gain by controlling the frequency and the dc electric field. Significant changes appear in the magnitude of the imaginary or gain by controlling the frequency and the dc electric field. Thus, we can tailor the intrinsic acoustic absorption large positive (or negative) values as the electric field is increased.

entering a thin slab of InSb (or another semiconductor) and if the dc electric field in the slab ensures acoustic gain as the sound traverses the slab in, say, the positive z direction. The basic requirement is to have a high enough applied electric field $E_0$ and to operate at ENZ conditions so that $|\text{Im}(k_1)|$ is high and damping is positive, $\text{Im}(k_1)/\text{Re}(k_1) > 0$ (refer to Figs. 2 and 3). It is important that the material surrounding the slab of length L is acoustically well matched to guarantee that a high portion of the incoming sound field enters the slab. In Fig. 4(a) we compute the transmission amplitude of an incoming plane wave impinging on a slab when a constant electric field is applied. The surrounding media are assumed to have an acoustic impedance $Z = 3 \times 10^7$ kg/(m²s), and the plasma frequency is $f = 2.69$ THz. Clearly, due to the exponential increase in the sound field along the positive z direction in a case with gain, the longer the slab is, the higher the transmission coefficient becomes. Calculations at different frequency values again show that amplification is strongest slightly below $\omega_p$. Since the real part of the wave number is considerably larger in magnitude than its imaginary part whenever gain is present, many oscillations in the acoustic field will take place over a slab length where gain is pronounced. This is a drawback, and instead, we propose a method for obtaining even larger gain for smaller slab structures by periodically switching the sign of the applied field $E_0$ in time with a period equal to $L\text{Re}(k)/\omega_0$, as depicted in Fig. 4(b). In doing so, the sound field experiences gain in propagating the slab in forward and backward directions since the sign of the damping term $\text{Im}(k)$ as seen in Fig. 4(c) opposes the sign of $E_0$ and is switched exactly when a wave round-trip is initiated. The effect of the controlled switching can be seen in Fig. 4(d), where the transmission $T$ and reflection $R$ coefficients are shown for cases with (s) and without switching.

FIG. 2. (Color online) Strong gain at ENZ conditions with frequencies around the plasmon frequency $f = 2.71$ THz for InSb. (a)–(d) The two wave numbers, $k_1$ and $k_2$, are captured as in Fig. 1 to demonstrate gain and absorption. Color bar units are in m⁻¹.

FIG. 3. (Color online) Dispersion relation and spectral dependence of the permittivity for InSb with $E_0 = -1000$ V/m. (a) The real parts of the wave numbers $k_1$ and $k_2$ are shown to be symmetric on a dispersion diagram. (b) Gain is present for $k_1$, as illustrated by $\text{Re}(k_1)/\text{Im}(k_1)$. (c) $\varepsilon/\varepsilon_0$ plotted near the plasmon frequency $f = 2.71$ THz. The dash-dotted line marks the spectral region where $\varepsilon = 0$. 

We now discuss a principle of an electroacoustic gain device based on a thin slab of semiconductor material controlled by an external dc electric field. It is evident that acoustic gain of a sound field is possible if a substantial part of the sound field
We have demonstrated how sound amplification in PZ materials can be substantially enhanced when operated at ENZ response. We utilized this finding by designing a switch-controlled optomechanical device producing larger-than-unity scattering coefficients of the radiated sound field, and we foresee that this technique will find many striking applications for sensing and spectroscopy. The fact that acoustic gain can be much higher than realized in the works by White and Hutson [10–12] makes the present idea less sensitive to crystal quality and further increases the potential for realizing applications. By envisioning that the present idea of sound amplification could be much higher than realized in the works by White and Hutson, we can foresee that this technique will find many striking applications for sensing and spectroscopy.

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